Homotopy quantum field theories

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Homotopy quantum field theories (HQFTs) generalize topological quantum field theories (TQFTs) by replacing manifolds by maps from manifolds to a fixed target space X. For example, any cohomology class in $H^3(X)$ defines a 3-dimensional HQFT with target X. If X is aspherical, that is $X = K(G, 1)$ for some group G, then this cohomological HQFT is related to the Dijkgraaf-Witten invariant and is computed as a Turaev-Viro state sum via the category of G-graded vector spaces. The general case is more involved. We will explain that if X has trivial second homotopy group, then the cohomological HQFT can be computed as a state sum via a monoidal 2-category (with non-trivial pentagonator) derived from the cohomology class and the k-invariant.