Combinatorics of faithfully balanced modules

Baptiste Rognerud

Faithfully balanced modules, also called modules with the double centralizer properties, appear in various places in the literature on ring theory, such as Schur-Weyl duality and Thrall's notion of a QF-1 algebra. The main known examples of faithfully balanced modules are faithful modules for a self-injective algebra, and more generally generators and tilting modules for any algebra. However, in general the behavior of faithfully balanced modules is rather mysterious.

In this talk I will illustrate this by discussing the combinatorics of faithfully balanced modules for the algebra of upper triangular n by n matrices. The theory extends known results about tilting modules, which are classified by binary trees, and counted with the Catalan numbers. The number of faithfully balanced modules is a 2-factorial number. Amongst them are $n!$ modules with exactly n indecomposable summands, which can be classified by interleaved binary trees or by increasing binary trees. This is part of joint work with William Crawley-Boevey, Biao Ma, and Julia Sauter in which we classify the faithfully balanced modules for Nakayama algebras.