

From a model structure on Frobenius categories to a prebifibration structure on exact categories

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Let \mathcal{E} be a weakly idempotent complete exact category with enough injective and projective objects. Assume that $\mathcal{M} \subseteq \mathcal{E}$ is a rigid, contravariantly finite subcategory of \mathcal{E} containing all the injective and projective objects, and stable under taking direct sums and summands. We show that \mathcal{E} is equipped with the structure of a prebifibration category with cofibrant replacements. As a corollary, we show, using the results of Demonet and Liu, that the category of finite presentation modules on the costable category $\overline{\mathcal{M}}$ is a localization of \mathcal{E} . These two corollaries are analogues for exact categories of results of Buan and Marsh that hold for triangulated categories.

If \mathcal{E} is a Frobenius exact category, we enhance its structure of prebifibration category to the structure of a model category (inspired from the case of triangulated categories made by Palu). This last result applies in particular when \mathcal{E} is any of the Hom-finite Frobenius categories appearing in relation to cluster algebras.