Categorical and Homotopical methods in Representation Theory

Abstracts

Karin Baur: CM-modules for Grassmannians

The category of Cohen-Macaulay modules over a quotient of a preprojective algebra is a cluster category associated to the coordinate ring of the Grassmannian. We study these categories, in particular in the tame cases. We show how one can associate SL_k-friezes to them. This is joint work with Bogdanic and Garcia Elsener and with Faber-Gratz-Serhiyenko-Todorov

Adrien Brochier: TFTs and Morita theory for (braided) tensor categories

The Baez-Dolan cobordism hypothesis, now a theorem, provides a dictionnary between topological field theories (TFT) and objects in higher categories satisfying strong finitness condition, generalizing finite-dimensionality for vector spaces. It turns out that many basic notions in representation theory have a natural interpretation in this context. A somewhat familiar example comes from the Morita theory of associative algebra, which can be encoded in a 2-category having algebras as objects, bimodules as morphisms, and morphisms of those as 2-morphisms. Constructions like Hochschild Homology, and properties like semi-simplicity, pop up spontaneously when using this 2-category as a target for TFTs. In this talk, I will explain a joint work with D. Jordan and N. Snyder replacing algebras by tensor and braided tensor categories. This leads to new examples of 3 and 4 dimensional TFTs, and organizes various important categorical structures and properties.

Bérénice Delcroix-Oger: Partition posets and species

Species of structure were introduced in 1980s by André Joyal. They provide a powerful tool to compute the action of the symmetric group on combinatorial structures. After introducing this notion, we will illustrate it with an application to combinatorial posets of partition types such as the 2-partition posets.

Ivo Dell'Ambroggio: Additive decomposition of the equivariant stable homotopy category

Let G be a finite group. The crossed Burnside ring of G, introduced by Yoshida, is a variant of the classical Burnside ring of finite G-sets. Working with p-local coefficients, in 2001 Bouc explicitly described the primitive idempotents of the crossed Burnside ring in terms of blocks of the group algebra, and showed that

the ring acts on the category of Mackey functors. He used this to describe the decomposition of the latter category into indecomposable factors. In this talk, I will explain how to use the recent theory of Mackey 2-functors (a categorified version of Mackey functors) and of Mackey 2-motives (an analogue in this context of Grothendieck's motives) in order to export the action of the crossed Burnside ring to obtain similar factor decompositions for many other additive categories "of G-equivariant objects". Most notably, this applies nicely to the G-equivariant stable homotopy category.

Loïc Foissy: Twisted Hopf algebras

A twisted Hopf algebra is a Hopf algebra in the category of linear species. The Fock functors allow to recover "classical" Hopf algebras from twisted ones. Numerous constructions and results can be lifted to the level of twisted bialgebras, such that cofreeness, shuffle and quasi-shuffles products, etc.

Using two tensor products in the category of species, we define the notion of cointeraction of twisted bialgebras. Examples on finite topologies and graphs allow to reconstruct Ehrahrt polynomials of chromatic polynomials in a canonical way from cointeraction of twisted bialgebras.

Lucie Jacquet-Malo: From a model structure on Frobenius categories to a prebifration structure on exact categories

Let $\mathcal{E}$ be a weakly idempotent complete exact category with enough injective and projective objects. Assume that $\mathcal{M} \subseteq \mathcal{E}$ is a rigid, contravariantly finite subcategory of $\mathcal{E}$ containing all the injective and projective objects, and stable under taking direct sums and summands. We show that $\mathcal{E}$ is equipped with the structure of a prefibration category with cofibrant replacements. As a corollary, we show, using the results of Demonet and Liu, that the category of finite presentation modules on the costable category $\overline{\mathcal{M}}$ is a localization of $\mathcal{E}$. These two corollaries are analogues for exact categories of results of Buan and Marsh that hold for triangulated categories.

If $\mathcal{E}$ is a Frobenius exact category, we enhance its structure of prefibration category to the structure of a model category (inspired from the case of triangulated categories made by Palu). This last result applies in particular when $\mathcal{E}$ is any of the Hom-finite Frobenius categories appearing in relation to cluster algebras.

Gustavo Jasso: Generalised BGP reflection functors via recollements

I will present a new approach to generalised BGP reflection functors which benefits of the additional flexibility provided by the language of stable infinity-categories. To this end, I will explain how the aforementioned language sheds new light on the classical concept of recollement and how to utilise the powerful methods afforded by the theory infinity-categories in our context. Familiarity with the theory of infty-categories will not be assumed.

This is a report on joint work with Tobias Dyckerhoff and Tashi Walde.

Sondre Kvamme: A generalization of the Nakayama functor

We introduce the notion of a Nakayama functor relative to an adjunction, generalizing the classical Nakayama functor for a finite-dimensional algebra. We show that it can be characterized in terms of an ambidextrous adjunction of monads and comonads. We also study this concept from the viewpoint of Gorenstein homological algebra, and in particular we obtain a generalization of the equality of the left and right injective dimension for a finite-dimensional Iwanaga-Gorenstein algebra. The talk will be illustrated on specific examples.

Baptiste Rognerud: Combinatorics of faithfully balanced modules

Faithfully balanced modules, also called modules with the double centralizer properties, appear in various places in the literature on ring theory, such as Schur-Weyl duality and Thrall's notion of a QF-1 algebra. The main known examples of faithfully balanced modules are faithful modules for a self-injective algebra, and more generally generators and tilting modules for any algebra. However, in general the behavior of faithfully balanced modules is rather mysterious.

In this talk I will illustrate this by discussing the combinatorics of faithfully balanced modules for the algebra of upper triangular n by n matrices. The theory extends known results about tilting modules, which are classified by binary trees, and counted with the Catalan numbers. The number of faithfully balanced modules is a 2-factorial number. Amongst them are $n!$ modules with exactly n indecomposable summands, which can be classified by interleaved binary trees or by increasing binary trees. This is part of joint work with William Crawley-Boevey, Biao Ma, and Julia Sauter in which we classify the faithfully balanced modules for Nakayama algebras.

Jérôme Scherer: Approximations of unbounded chain complexes by truncations

This is joint work with Amnon Neeman, Wojciech Chacholski, and Wolfgang Pitsch. We package Spaltenstein's original idea to build resolutions of unbounded chain complexes in $Ch(R)$ into a pair of adjoint functors which form what we call a model approximation: The model category of towers of truncated chain complexes approximate the category $Ch(R)$. We then extend these methods to do relative homological algebra. We will also present certain rings and well chosen (or rather badly chosen?) classes of injectives for which our methods do not work as smoothly as expected.

Radu Stancu: Fusion Stable Burnside Rings

Let p be a prime. Fusion systems on a finite p-group S are a generalisation of the p-local structure given by a finite group G on S, when S is a subgroup of G. For a fusion system $\mathcal{F}$ on S there are associated stable Burnside rings. When $\mathcal{F}$ is saturated - corresponding to the situation where S is a Sylow p-subgroup of G - the associated stable Burnside rings possess very strong properties. For

example, the monoide of $\mathcal{F}$-stable S-sets is free. We show that this is not the case outside of the saturated world and we look for families of p-groups where some of these properties are still true. The talk presents a joint work - in progress - with Aktham Mulla Mohamad.

Alexis Virelizier: Homotopy quantum field theories

Homotopy quantum field theories (HQFTs) generalize topological quantum field theories (TQFTs) by replacing manifolds by maps from manifolds to a fixed target space X. For example, any cohomology class in $H^3(X)$ defines a 3-dimensional HQFT with target X. If X is aspherical, that is $X = K(G, 1)$ for some group G, then this cohomological HQFT is related to the Dijkgraaf-Witten invariant and is computed as a Turaev-Viro state sum via the category of G-graded vector spaces. The general case is more involved. We will explain that if X has trivial second homotopy group, then the cohomological HQFT can be computed as a state sum via a monoidal 2-category (with non-trivial pentagonator) derived from the cohomology class and the k-invariant.